

Math 3070/6070 Homework 1  
Due: Sept 1st, 2025  
Submit to your TA at Recitations

1. (1.1) For each of the following experiments, describe the sample space.
  1. Toss a coin four times.
  2. Count the number of insect-damaged leaves on a plant.
  3. Measure the lifetime (in hours) of a particular brand of light bulb.
  4. Record the weights of 10-day-old rats.
  5. Observe the proportion of defectives in a shipment of electronic components.
2. (1.5) Approximately one-third of all human twins are identical (one-egg) and two-thirds are fraternal (two-egg) twins. Identical twins are necessarily the same sex, with male and female being equally likely. Among fraternal twins, approximately one-fourth are both female, one-fourth are both male, and half are one male and one female. Finally, among all U.S. births, approximately 1 in 90 is a twin birth. Define the following events:

$$\begin{aligned}A &= \{\text{a U.S. birth results in twin females}\} \\B &= \{\text{a U.S. birth results in identical twins}\} \\C &= \{\text{a U.S. birth results in twins}\}\end{aligned}$$

1. State, in words, the event  $A \cap B \cap C$ .
  2. Find  $\Pr(A \cap B \cap C)$ .
3. (1.6) Two pennies, one with  $\Pr(\text{head}) = u$  and one with  $\Pr(\text{head}) = w$ , are to be tossed together independently. Define

$$\begin{aligned}p_0 &= \Pr(0 \text{ heads occur}), \\p_1 &= \Pr(1 \text{ head occurs}), \\p_2 &= \Pr(2 \text{ heads occur}).\end{aligned}$$

Can  $u$  and  $w$  be chosen such that  $p_0 = p_1 = p_2$ ? Prove your answer.

4. (1.13) If  $\Pr(A) = \frac{1}{3}$  and  $\Pr(B^c) = \frac{1}{4}$ , can  $A$  and  $B$  be disjoint? Explain.
5. (1.27) Verify the following identities for  $n \geq 2$ .

1.  $\sum_{k=0}^n (-1)^k \binom{n}{k} = 0$
2.  $\sum_{k=1}^n k \binom{n}{k} = n2^{n-1}$
3.  $\sum_{k=1}^n (-1)^{k+1} k \binom{n}{k} = 0$