

Math 3070/6070 Homework 5

Due: Nov 17th, 2025

1. (3.15) In class, we showed that the *Poisson*(λ) distribution is the limit of the negative binomial(r, p) distribution as $r \rightarrow \infty, p \rightarrow 1, r(1-p) \rightarrow \lambda$. Show that under these conditions the mgf of the negative binomial converges to that of the Poisson.

2. (3.23) The *Pareto distribution*, with parameters α and β , has pdf

$$f(x) = \frac{\beta \alpha^\beta}{x^{\beta+1}}, \alpha < x < \infty, \alpha > 0, \beta > 0.$$

1. Verify that $f(x)$ is a pdf.
 2. Derive the mean and variance of this distribution.
 3. Prove that the variance does not exist if $\beta \leq 2$.
3. (3.28) Show that each of the following families is an exponential family
 1. normal family with either parameter μ or σ known.
 2. gamma family with either parameter a or b known or both unknown.
 3. beta family with either parameter a or b known or both unknown.
 4. Poisson family
 5. negative binomial family with r known, $0 < p < 1$.
 4. (3.33) For each of the following families:
 1. Verify that it is an exponential family.
 2. Describe the curve on which the θ parameter vector lies.
 3. Sketch a graph of the curved parameter space.
 - (a) $n(\theta, \theta)$
 - (b) $n(\theta, a\theta^2)$, a known
 5. (3.37) Show that if $f(x)$ is a pdf, symmetric about 0, then μ is the median of the location-scale pdf

$$(1/\sigma)f((x-\mu)/\sigma), -\infty < x < \infty.$$