

Math 3070/6070 Homework 6

Due: No submission

1. (4.1) A random point (X, Y) is distributed uniformly on the square with vertices $(1, 1)$, $(1, -1)$, $(-1, 1)$, and $(-1, -1)$. That is, the joint pdf is $f(x, y) = \frac{1}{4}$ on the square. Determine the probabilities of the following events.

1. $X^2 + Y^2 < 1$
2. $2X - Y > 0$
3. $|X + Y| < 2$

2. (4.5)

1. Find $\Pr(X > \sqrt{Y})$ if X and Y are jointly distributed with pdf

$$f(x, y) = x + y, \quad 0 \leq x \leq 1, \quad 0 \leq y \leq 1$$

2. Find $\Pr(X^2 < Y < X)$ if X and Y are jointly distributed with pdf

$$f(x, y) = 2x, \quad 0 \leq x \leq 1, \quad 0 \leq y \leq 1$$

3. (4.11) Let U = the number of trials needed to get the first head and V = the number of trials needed to get two heads in repeated tosses of a fair coin. Are U and V independent random variables?
4. (4.17) Let X be an exponential(1) random variable, and define Y to be the integer part of $X + 1$, that is

$$Y = i + 1 \quad \text{if and only if} \quad i \leq X < i + 1, \quad i = 0, 1, 2, \dots$$

1. Find the distribution of Y . What well-known distribution does Y have?
 2. Find the conditional distribution of $X - 4$ given $Y \geq 5$.
5. (4.20) X_1 and X_2 are independent $N(0, \sigma^2)$ random variables.

1. Find the joint distribution of Y_1 and Y_2 , where

$$Y_1 = X_1^2 + X_2^2 \quad \text{and} \quad Y_2 = \frac{X_1}{\sqrt{Y_1}}$$

2. Show that Y_1 and Y_2 are independent.